[7] $y = \sinh^{-1} x$ IF AND ONLY IF $x = \sinh y$

$\neq \dfrac{1}{\sinh x}$

$f^{-1} \neq \dfrac{1}{f}$

NAME OF FUNCTION

$=$ INVERSE OF FUNCTION, NOT RECIPROCA

[d][iii] $y = \sinh^{-1} x$

$$x = \sinh y$$

$$x = \frac{e^y - e^{-y}}{2}$$

$$2x = e^y - e^{-y}$$

$$2x = e^y - \frac{1}{e^y} \qquad \text{LET } z = e^y$$

$$z(2x) = \left(z - \frac{1}{z}\right)z$$

$$2xz = z^2 - 1$$

$$0 = z^2 - 2xz - 1$$

USE QUADRATIC FORMULA

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = -2x$ (COEFFICIENT OF z)

$c = -1$

$$z = \frac{2x \pm \sqrt{4x^2 - 4 \cdot 1 \cdot -1}}{2}$$

$$z = \frac{2x \pm \sqrt{4x^2 + 4}}{2}$$

$$z = \frac{2x \pm \sqrt{4(x^2+1)}}{2}$$

>0

$$e^y = z = \frac{2x \pm 2\sqrt{x^2+1}}{2} = x \pm \sqrt{x^2+1} = x + \sqrt{x^2+1}$$

SINCE $x - \sqrt{x^2+1} < 0$

eg. $5 - \sqrt{5^2+1} = 5 - \sqrt{26} < 0$

$$e^y = x + \sqrt{x^2+1}$$

$$y = \ln\left(x + \sqrt{x^2+1}\right)$$

$$\boxed{\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)}$$

$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$

[6] YESTERDAY WE SOLVED $\sinh x = 1 \rightarrow x = \ln(1+\sqrt{2})$

$\sinh^{-1} 1 = x$

$\sinh^{-1} 1 = \ln\left(1 + \sqrt{1^2+1}\right)$

OUR NEW FORMULA FOR $\sinh^{-1} x$

IS CONSISTENT WITH YESTERDAY'S WORK

☆ THIS DOESN'T PROVE THE FORMULA

IS CORRECT FOR ALL x

$f(f^{-1}(x)) = x$

$f^{-1}(f(x)) = x$

TO PROVE THAT $g(x) = f^{-1}(x)$

MATH 41: (PRECALC 1)

NEED TO SHOW

$f(g(x)) = x = g(f(x))$

[7]

IE. NEED TO PROVE $\sinh(\sinh^{-1} x) = x$ [d][i]

AND $\sinh^{-1}(\sinh x) = x$ [d][ii]

[7][d][i] PROVE $\sinh(\sinh^{-1}x) = x$.

$\sinh^{-1}x$
$= \ln(x+\sqrt{x^2+1})$

$$\sinh(\sinh^{-1}x) = \frac{e^{\sinh^{-1}x} - e^{-\sinh^{-1}x}}{2}$$

$$= \frac{e^{\ln(x+\sqrt{x^2+1})} - e^{-\ln(x+\sqrt{x^2+1})}}{2}$$

$$= \frac{\left(x+\sqrt{x^2+1}\right) - \frac{1}{x+\sqrt{x^2+1}}}{2}$$

$$\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}}$$

$(A+B)^2$
$= A^2+2AB+B^2$

$$= \frac{(x+\sqrt{x^2+1})^2 - 1}{2(x+\sqrt{x^2+1})}$$

$$= \frac{x^2+2x\sqrt{x^2+1} + x^2 \cancel{+1} \cancel{-1}}{2(x+\sqrt{x^2+1})}$$

$$= \frac{2x^2+2x\sqrt{x^2+1}}{2(x+\sqrt{x^2+1})}$$

$$= \frac{\cancel{2}x\cancel{(x+\sqrt{x^2+1})}}{\cancel{2}\cancel{(x+\sqrt{x^2+1})}} = x$$

SO,
$\sinh(\sinh^{-1}x) = x$

[7][d][ii] PROVE $\sinh^{-1}(\sinh x) = x$

$$\sinh^{-1}(\sinh x)$$
$$= \ln\left(\sinh x + \sqrt{\sinh^2 x + 1}\right)$$
$$= \ln\left(\sinh x + \sqrt{\cosh^2 x}\right)$$
$$= \ln(\sinh x + |\cosh x|)$$
$$= \ln(\sinh x + \cosh x)$$
$$= \ln e^x \quad = \log_e e^x$$
$$= x$$

so, $\sinh^{-1}(\sinh x) = x$

so $\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$

FROM HOMEWORK

[4][a][ii]

$\cosh^2 x = 1 + \sinh^2 x$

$\sqrt{y^2} = |y|$

(NOT JUST y)

[0][d]

$\cosh x > 0$

FOR ALL x

[0][h]

$\cosh x + \sinh x$

$= e^x$

10.7 POLAR COORDINATES

RECTANGULAR COORDINATES

(x, y)

eg. $(x, y) = (1, 3)$